

A NEW DEVELOPMENT IN Z-TRANSFORM MODELLING OF TRANSFORMERS

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Abstract

Transformers have complicated winding structures, which cannot be simulated by simple R, L and C circuits for electromagnetic transient analysis. The conventional transformer models can only be used for 50Hz and low frequency analysis. More accurate models, such as Z-transform models, should be used if the frequency involved in a transient is above 5kHz. In this paper, a further development in Z-transform modelling is presented. The new method developed can be used to obtain the transformation matrix from an asymmetric admittance matrices. Testing and calculation results on a three-phase transformer have proved the accuracy of this method.

1. INTRODUCTION

Transformers are made to operate at the frequency of 50/60 Hz. In the study of the transient characteristics of a transformer, high frequency simulations are used to determine the terminal overvoltages caused by circuit switching and lightning. The conventional methods of using the equipment's leakage impedance in these simulations can lead to misleading results as this only works for lower frequency range. Furthermore, the transformer's winding structures are very complicated and the simulation of using concentrated capacitance may not fully model the real situation. A z-transform model of the transformer has been studied over the years[1,2] and it has been found that the equipment's transient characteristics can be accurately determined using this model.

In order to obtain the mathematical model, tests are conducted onto a chosen transformer and the data obtained are converted into various forms. One of the stages in the process is to obtain the transformation matrix. This stage is important because it allows the experimenter to break down an n-port network to a series of n one-port networks. In other words, it allows the simplification of a 6-port network to 6 one-port networks. In this research, the 6-port network is used to represent the chosen transformer, as the transformer used is a 3-phase 2-winding transformer. This is done to equate and simplify the equipment into its basic components so that further analysis can be conducted.

In Z-transform modelling, the transformer's transformation matrix must be obtained in order to transform the admittance matrix to a diagonal matrix. If the transformation matrix is frequency dependant, the Z-transform model will be too complicated making the calculation method very tedious. For a

symmetrical transformer admittance matrix, a transformation matrix can always be derived to transform the admittance matrix to a diagonal matrix leading to three separated-mode models. This can significantly simplify the calculation. However, it has been a problem for asymmetric admittance matrices to be transformed to diagonal matrices. Further investigation should therefore carried out.

2. THEORY

The terminal voltages $V(s)$ and currents $I(s)$ of a transformer are related by :

$$I(s) = Y(s).V(s) \quad \dots(1)$$

where $Y(s)$ is the transformer's admittance.

The high frequency transformer model can be determined accurately when the transformer's entry impedance is used. When impulses are applied, the measured t-domain voltages and currents are transformed to the s-domain and the entry impedance $Z(s)$ and admittance $Y(s)$ are respectively :

$$Z(s) = \frac{V(s)}{I(s)} \quad \text{and} \quad Y(s) = \frac{I(s)}{V(s)} \quad \dots(2)$$

where $V(s) = F[v(t)]$ and $I(s) = F[I(t)]$.

After obtaining the frequency domain quantities, $Y(s)$ can be synthesized using the multi-product rational function to

$$Y(s) = A \prod_{k=1}^n \frac{A_k s^2 + B_k s + 1}{C_k s^2 + D_k s + 1} \quad \dots(3)$$

The coefficients of $Y(s)$ can be determined by minimizing the error function

$$Q(q^t) = \sum_{i=1}^L W(\omega_i) [|Y(\omega_i)| - |Y(j\omega_i)|]^2 \dots(4)$$

where $q^t = A, A_1, B_1, C_1, D_1, \dots, A_n, B_n, C_n, D_n, W(\omega_i) =$ frequency weighting function and $L =$ sampling number in the frequency domain.

To transform the quantities into the z-domain, the following relation is used :

$$z = \exp [s\Delta t] \dots(5)$$

Equation (4) can be approximated to become this bilinear transformation :

$$s = \frac{2}{\Delta t} \cdot \frac{1-z^{-1}}{1+z^{-1}} \dots(6)$$

Substituting equation (6) into equation (3), $Y(s)$ can be transformed to the z-plane and the result is

$$Y(s) = \frac{Y_0 + \sum_{k=1}^N a_k Z^{-k}}{1 + \sum_{k=1}^N b_k Z^{-k}} \dots(7)$$

The z-domain quantities are later transformed back to the t-domain, giving :

$$I(n) = Y_0 V(n) + I_p(p-n) \dots(8)$$

where $I_p(p-n) = \sum_{k=1}^N (a_k V(n-k) - b_k I(n-k))$

An equivalent circuit diagram represented by equation (8) is shown in Fig.1^[2].

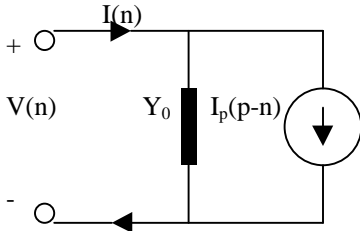


Fig. 1 The Equivalent Circuit For The Transformer's Entry Admittance Model

As mentioned before, one of the task of the project is to break down an n-port network into an equivalent of n one-port networks. This can be done as the transformer's parameters are basically represented in the form of matrices. Using $[I] = [Y].[V]$ or

$$\begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & & Y_{2n} \\ \vdots & & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_n \end{bmatrix} \dots(9)$$

where $[Y]$ is the transformer's admittance matrix, and $Y_{AA} = I_A/V_A, Y_{BA} = I_B/V_A, Y_{CA} = I_C/V_A, \dots$

Equation (9) can be equated to $[I] = [Y_m].[V]$:

$$\begin{bmatrix} I_{m1} \\ \vdots \\ I_{mn} \end{bmatrix} = \begin{bmatrix} Y_{m1} & 0 & \dots & 0 & 0 \\ 0 & Y_{m2} & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & Y_{mn} \end{bmatrix} \begin{bmatrix} V_{m1} \\ \vdots \\ V_{mn} \end{bmatrix} \dots(10)$$

where $[Y_m]$ is the diagonal mode admittance matrix.

From equation (10), $I_{m1} = Y_{m1} \cdot V_{m1}, I_{m2} = Y_{m2} \cdot V_{m2}, \dots, I_{mn} = Y_{mn} \cdot V_{mn}$ and so on. This is the basic form where the n-port network is simplified to n one-port networks, as shown below by a 3-port network :

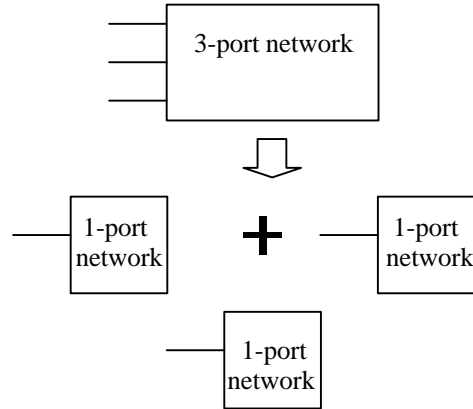


Fig. 2 Transformation Of A 3-Port Network To Three 1-Port Networks

To convert $[Y]$ to $[Y_m]$, the following relation is used :

$$[Y] = [Q]^{-1}[Y_m][Q] \dots(11)$$

where $[Q]$ is the transformation matrix that is used to diagonalise the admittance matrix to the mode admittance matrix.

An ideal transformer, which consists only of resistances, inductances and capacitances, coupled with the reciprocity of ideal transformers, the admittance matrix should be symmetrical. However, the admittance matrix in practical transformers is

slightly asymmetrical but the degree of asymmetriness is very low. The conventional mathematical tools such as the method of single value decomposition can be used to diagonalise the admittance matrix to mode admittance matrix. In the research, both the admittance and mode admittance matrices are found and the transformation matrix is to be determined. The ease of obtaining the transformation matrix is also proportional to the degree of symmetry of the admittance matrix. Therefore, this could pose a great problem if admittance matrix is found to be severely asymmetric.

3. DERIVATION OF SYMMETRIC MATRIX

The transformation matrix [Q] was defined earlier in equation (11). Rearranging the equation, the following relation is obtained :

$$[Q] \cdot [Y] = [Y_m] \cdot [Q] \quad \dots(12)$$

[Q], [Y] and [Y_m] are all n × n square matrices. To obtain [Q], the solving of n² unknowns with n² linear equations is done. Equation (12) can be solved manually if the order of the matrix is small. However, as seen earlier, the number of equations needed is equivalent the square of the order, which make the solving of the matrix a tedious task.

The above equation can be easily solved by using engineering softwares such as Mathematica or Matlab. However, if equation (12) is entered, these softwares will generate a null matrix for the transformation matrix. This is because equation (12) forms

$$\underline{A} \cdot \underline{x} = \underline{0} \quad \dots(13)$$

To solve the above matrix, all the answers generated will be null matrices. The reason behind this is that there are not sufficient conditions given to solve equation (13). In other words, this is a numerical problem and a boundary condition must be applied in order for non-zero answers to be generated.

Equation (12) is however a valid equation but it is difficult to obtain a specific boundary condition. In order to solve this problem, equation (13) is modified to give

$$\underline{A} \cdot \underline{x} = \underline{\epsilon} \quad \dots(14)$$

where $\epsilon \rightarrow 0$. When the modified equation (14) is re-entered into Mathematica, a non-zero transformation matrix is generated.

The transformer's transformation matrix is a constant by theory. However, when the frequency increases, the element values of matrices [Y] and [Y_m] will decrease. Therefore, the element values for the transformation matrix will increase and the error will

be large. This is because the value of ϵ used is constant for all frequencies, i.e. ϵ is frequency independent. Therefore, the value of boundary conditions should be revised.

The next step after the approximation of the boundary condition is to improve the 'quality' of the transformation matrix. This has been done by making the value of ϵ to be frequency dependent. The method used the simple technique of proportionality. When the frequency increases, the terminal currents were observed to decrease linearly. This is however not true in other cases where the decrease may be exponential in nature.

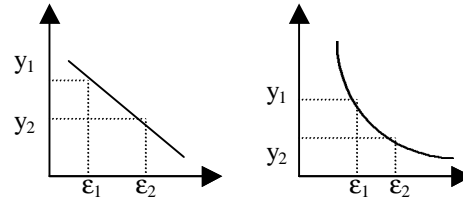


Fig. 3 The Values Of ϵ Used Depends On The Decreasing Pattern Of The Admittance Values

Therefore, the value of ϵ used is approximated by observing the decreasing pattern of the transformer admittance values. This is then entered into Mathematica and the transformation matrices generated have reasonably small errors. Therefore, equation (14) can be rewritten as :

$$\underline{A}_f \cdot \underline{x}_f = \underline{\epsilon}_f \quad \dots(15)$$

where all the quantities above are frequency dependent. After that, the entire equation is divided by ϵ . Since $\underline{A}_f \cdot \underline{x}_f$ is a lot larger than $\underline{\epsilon}_f$ after the division, the equation can be reverted back to become the original equation again, as was shown by equation (13).

$$\begin{aligned} & \underline{A}_f \cdot \underline{x}_f / \epsilon = \underline{\epsilon}_f / \epsilon \\ \Rightarrow & \underline{A}_f \cdot \underline{x}_f / \epsilon = \underline{1} \\ \Rightarrow & \approx \underline{A}_f \cdot \underline{x}_f / \epsilon = \underline{0} \\ \Rightarrow & \underline{A}_f \cdot \underline{x}_f = \underline{0} \end{aligned}$$

as $\underline{A}_f \cdot \underline{x}_f / \epsilon \gg \underline{1}$. The transformation matrix can then be averaged out and this is taken to be the frequency independent transformation matrix, a characteristic of the transformer itself.

4. EXPERIMENTAL RESULTS

The first step of the project was to conduct a low voltage test onto a chosen 10kVA transformer. A voltage source is applied to a terminal while the rest of

the terminals are grounded. The currents in all other terminals are then measured. The test is repeated by applying the source at all the terminals and varying the frequency of the applied voltage source, which in this case is 0.15V. Below are the results when the voltage source is applied at primary terminal 'A'.

Table 1 The Currents Measured When Voltage Applied To Terminal 'A'

F / Hz	Primary Windings			Secondary Windings		
	I _a / A	I _b / A	I _c / A	I _a / A	I _b / A	I _c / A
50	2	1	1.2	1.6	0.8	0.8
100	1.4	0.8	1	1.2	0.8	0.7
200	1.1	0.8	0.8	1	0.6	0.6
300	0.8	0.6	0.6	0.8	0.6	0.6
400	0.55	0.5	0.5	0.5	0.5	0.5
500	0.4	0.4	0.4	0.4	0.3	0.3
1,000	0.16	0.1	0.12	0.12	0.08	0.08
1,500	0.14	0.09	0.1	0.12	0.08	0.07
2,000	0.12	0.09	0.1	0.1	0.08	0.07
2,500	0.1	0.09	0.09	0.1	0.07	0.07
3,000	0.09	0.08	0.08	0.09	0.07	0.07
3,500	0.09	0.08	0.08	0.09	0.07	0.06
4,000	0.09	0.08	0.08	0.09	0.06	0.06
4,500	0.08	0.08	0.07	0.08	0.06	0.05
5,000	0.06	0.06	0.06	0.05	0.04	0.04
7,500	0.025	0.012	0.018	0.02	0.01	0.01
10,000	0.02	0.011	0.015	0.016	0.009	0.009
12,500	0.016	0.01	0.013	0.013	0.008	0.008
15,000	0.012	0.008	0.01	0.012	0.007	0.007
17,500	0.01	0.007	0.01	0.01	0.007	0.007
20,000	0.01	0.007	0.008	0.008	0.007	0.006

It can be seen that as frequency increases, the currents will also decrease but at high frequencies, the currents converge.

The next step is to obtain the transformer's admittance and the diagonal mode admittance matrices, as defined by equations (9) and (10). The admittance and mode admittance matrices for frequency 50 Hz are :

Table 2 50 Hz Admittance Matrix

13.3333	8	8	9.33333	6.66667	10.6667
6.66667	10.6667	8.66667	5.33333	10	5.33333
8	8	11.3333	8.66667	7.33333	9.33333
10.6667	9.33333	9.33333	10	7.33333	6.66667
5.33333	9.33333	9.33333	8	10.6667	7.33333
5.33333	10	10.6667	9.33333	7.33333	10.6667

Table 3 50 Hz Mode Admittance Matrix

56	0	0	0	0	0
0	46.6667	0	0	0	0
0	0	52.6667	0	0	0
0	0	0	53.3333	0	0
0	0	0	0	50	0
0	0	0	0	0	53.3333

As before, when frequency increases, the element values of the matrices decrease.

After obtaining the matrices, the transformation matrix is found. The theory of this is written under Section 3. Initially, the element values of the transformation matrix found was unacceptable, as shown below :

Table 3 The Initial Transformation Matrix

-0.199255	0.176913	0.090602	-0.249239	-0.246732	-0.978892
0.176913	0.521898	0.450637	0.210864	0.099077	-0.752669
0.090602	0.450637	0.378530	0.033506	-0.022260	-0.728776
-0.249239	0.210864	0.033506	-0.300513	-0.574752	-1.312320
-0.246732	0.099077	-0.022260	-0.574752	-0.409117	0.663971
-0.978892	-0.752669	-0.728776	-1.312320	0.663971	-1.780290

However, when the improvement stage taken, the element values have improved and the results are shown below :

Table 4 The Final Transformation Matrix For All Frequencies

-0.199255	0.500122	0.342024	-0.342101	-0.344689	-1.80884
-0.146297	0.521898	0.353471	-0.131725	-0.366171	-2.08730
-0.160821	0.547794	0.378530	-0.323274	-0.384769	-1.78988
-0.156376	0.553452	0.390286	-0.300513	-0.415074	-1.84992
-0.148774	0.564324	0.340249	-0.734429	-0.409117	-1.72012
-0.148944	0.581963	0.332329	-0.774717	-0.392179	-1.78029

The transformation matrix can then be improved when the actual function of the plots as shown in Figure 3 is known. In that way, the values of ϵ can be more accurately determined.

Next, the mode voltages and currents are determined by using the following equation :

$$[Q].[I] = [Y_m].[Q].[V]$$

$$\Rightarrow [I_m] = [Y_m].[V_m] \quad \dots(16)$$

The 50 Hz quantities are shown below :

$$\begin{bmatrix} -15.2604 \\ -15.2817 \\ -14.3170 \\ -14.6727 \\ -17.3162 \\ -17.9334 \end{bmatrix} = \begin{bmatrix} 56 & 0 & 0 & 0 & 0 & 0 \\ 0 & 46.6667 & 0 & 0 & 0 & 0 \\ 0 & 0 & 52.6667 & 0 & 0 & 0 \\ 0 & 0 & 0 & 53.3333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 53.3333 \end{bmatrix} \begin{bmatrix} -0.277851 \\ -0.278419 \\ -0.259863 \\ -0.266722 \\ -0.316180 \\ -0.327276 \end{bmatrix}$$

It can be seen that the approximation is quite reasonable.

The transformer's mode admittance are then transformed from the f-plane to the s-plane by plotting the mode admittances against log(frequency). The formula and a typical plot is shown below :

$$y_m(f) \Rightarrow y_m(s) = \frac{y_0(s-a_1)(s-a_2)...(s-a_n)}{(s-b_1)(s-b_2)...(s-b_n)} \quad \dots(17)$$

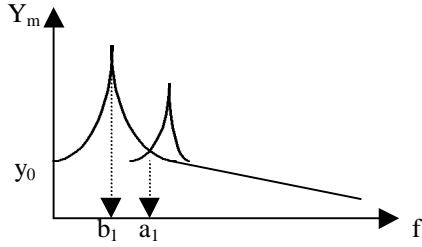


Fig. 4 Plot Of Mode Admittance vs. Frequency

There are four peaks in the plot and therefore there are four values of b_n and three values of a_n . For primary terminal 'A',

$$y_{m1}(s) = \frac{23.5(s-92.5)(s-160)(s-235)}{(s-50)(s-100)(s-200)(s-300)}$$

The mode admittance is next transformed from the s-plane to the z-plane using the formula defined by equation (7) and for the primary terminal 'A',

$$I_{m1}(z) = \frac{23.5 + 92.5z^{-1} + 160z^{-2} + 235z^{-3}}{1 + 50z^{-1} + 100z^{-2} + 200z^{-3} + 300z^{-4}}$$

Finally, the quantities are transformed back to the t-plane using equation (8) and later simplified to the following equations :

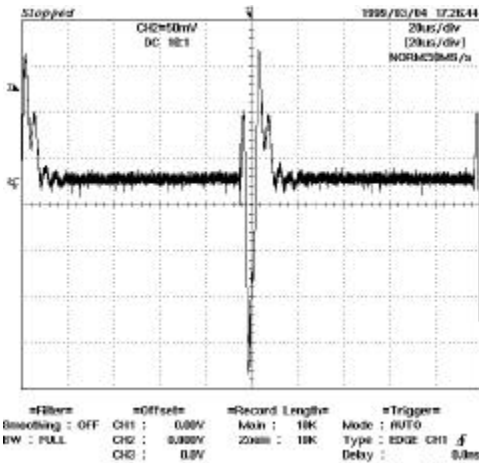
$$I_m(t) + \sum_{j=1}^n b_j I_m(t - \Delta t) = y_0 V_m(t) + \sum_{k=1}^n a_k V_m(t - \Delta t)$$

$$\Rightarrow I_m(t) = y_0 V_m(t) + \left[\sum_{k=1}^n a_k V_m(t - k\Delta t) - \sum_{j=1}^n b_j I_m(t - j\Delta t) \right]$$

...(18)

The transformer's primary terminal 'A' finally takes the form :

$$I_{m2}(t) = 19.5V_{m2}(t) + \{ [82.5V_{m2}(t - \Delta t) + 140V_{m2}(t - 2\Delta t) + 195V_{m2}(t - 3\Delta t)] - [50I_{m2}(t - \Delta t) + 100I_{m2}(t - 2\Delta t) + 200I_{m2}(t - 3\Delta t) + 300I_{m2}(t - 4\Delta t)] \}$$



The impulse response of the primary terminal 'A' when an impulse is applied at secondary terminal 'a' is as shown in Fig.5.

Fig. 5 Impulse Response At Primary Terminal 'A' When The Impulse Is Applied At Secondary Terminal 'a'

The impulse response shown above is represented by the t-plane equation that was derived earlier.

5. CONCLUSIONS

A high frequency transient analysis was conducted onto a chosen transformer and the procedures and results have been described.

Generally, it can be seen that as frequency increases, the measured quantities will decrease accordingly. The main purpose is to obtain the transformation matrix of the transformer. This is however a difficult task as the admittance matrices found were severely asymmetrical. A method has been developed to obtain the transformation matrix. Initially, the element values of the transformation matrix were totally below satisfactory level and to elevate this problem, an improvement step was made to improve the quality of the results. After this improvement step, the final results were found to have improved significantly.

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