Module 2: Transformations and Scene Creation

Linear Algebra Review Sheet

1. Let $a = (1, 2, 3), \quad b = (-1, 4, 0), \quad c = (3, -2, 4)$. Compute the following:
    a. $a \bullet a$
    b. $\|a\|^2$
    c. $a \bullet b$
    d. $b \bullet a$
    e. $a \bullet (b + c)$
    f. $a \bullet b + a \bullet c$

2. Find $2u + 5(v + w)$ where $u = (1, 7, -2)$, $v = (3, -1, 0)$, $w = (-2, 2, -2)$.

3. Construct the vector with unit length given by $\dfrac{v}{\|v\|}$, where $v = (2, -4, 3)$.

4. Calculate $u \cdot (v + w)$, where $u = (1, 2, 2)$, $v = (-3, 0, -2)$, $w = (-1, -1, 4)$.

5. Find the angle between the vectors $a = (2, 2, -1)$ and $b = (5, -4, 2)$.

6. Find two unit vectors orthogonal to both $(1, 1, \frac{1}{2})$ and $(2, 0, 1)$.

7. Find the projection of *w* onto *v*, where $w = (2, 3, 1)$, $v = (3, 0, 0)$, as well as the associated orthogonal vector *u*.

8. What is the vector that is perpendicular to both $u = (1, 2, 3)$ and $v = (-1, 4, -2)$? Use the cross-product operator.

9. What is the area of the parallelogram with sides *u* and *v* for $u = (1, 2, 3)$ and $v = (-1, 4, -2)$? You need to compute $\|u \times v\|$.

10. For $u = (1, 2, 0)$ and $v = (3, a, 1)$, what value of *a* is required for *u* and *v* to be perpendicular?

11. Calculate the unit normal to the plane given by $2x - 3y + 4z = 5$.

12. Find the normal to the plane that passes through the points $A = (1, 2, 1), \quad B = (-3, -1, 2), \quad C = (1, -1, -1)$. Hint: find the cross-product $(B - A) \times (C - A)$.

13. Find the equation of the plane in the preceding question. Hint: use the equation $n \bullet (P - P_0) = 0$ where $P = (x, y, z)$ and P_0 is one of the specified points.

14. Two planes are parallel if their normal vectors are parallel. If they are not parallel, they meet in a straight line and the angle between them is the acute angle between their normal vectors. If the two normal vectors are orthogonal, the planes are perpendicular. Are the following planes parallel, perpendicular or neither? If neither, compute the angle between them (using $\cos\theta = \frac{n_1 \bullet n_2}{\|n_1\| \|n_2\|}$).

    a. $x + y + z = 3, \quad 2x - y - z = 2$
    b. $2x + y - z = 4, \quad 2x - 3y - 2z = 2$.

15. Calculate the following matrix products:

$$(a) \quad \begin{pmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} -2 & 3 & 2 \\ 1 & 4 & -3 \\ 5 & 0 & 1 \end{pmatrix}$$

$$(b) \quad \begin{pmatrix} 3 & 0 & 0 & 1 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & 1 & -2 \\ 2 & -1 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

16. Which matrix (M_1, M_2, M_3) is the inverse of $M = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 3 & 0 \\ 0 & 1 & 4 \end{pmatrix}$?

$$M_1 = \begin{pmatrix} \frac{12}{25} & \frac{1}{50} & -\frac{3}{25} \\ -\frac{4}{25} & \frac{4}{25} & \frac{1}{25} \\ \frac{1}{25} & -\frac{1}{25} & \frac{6}{25} \end{pmatrix}, \quad M_2 = \begin{pmatrix} \frac{12}{25} & \frac{1}{25} & -\frac{3}{25} \\ -\frac{4}{25} & \frac{8}{25} & \frac{1}{25} \\ \frac{1}{25} & -\frac{2}{25} & \frac{6}{25} \end{pmatrix}, \quad or \quad M_3 = \begin{pmatrix} \frac{12}{25} & \frac{1}{25} & -\frac{3}{25} \\ -\frac{4}{25} & \frac{8}{25} & \frac{1}{25} \\ \frac{1}{25} & -\frac{2}{25} & \frac{4}{25} \end{pmatrix}.$$

17. Demonstrate that a rotation followed by a translation is not the same as a translation followed by a rotation, for the particular example given by a rotation of 30 degrees around the *z*-axis and a translation of 1, 2 and –1 in the *x*, *y* and *z*-directions respectively. In other words, form the *R* and *T* matrices, and compute *RT* and *TR*.