The University of Queensland
School of Information Technology and Electrical Engineering

Semester Two, 2009

COMS3200/COMS7201 – Tutorial 4 - Solutions

Question 1

Television channels are 6MHz wide. What's the maximum bit-rate (bps) which can be sent if four-level digital signals are used? Assume a noiseless channel.

From Nyquist's theorem:

$$\begin{aligned}\text{Max Data Rate} &= 2W\log_2 M \\ &= 2\times 6000000\times\log_2 4 \\ &= 24\text{Mbps}\end{aligned}$$

Question 2

If a binary signal is sent over a 3kHz bandwidth channel whose signal to noise ratio is 20dB, what is the maximum achievable data rate?

In this question we are given *both* the number of levels in the signal (binary, i.e. 2) and the signal-to-noise ratio of the channel. We therefore need to consider both the Nyquist limit and the Shannon limit and take the lesser value as the answer.

From Shannon's theorem:

$$\text{Max Data Rate} = W\log_2\left(1+\frac{S}{N}\right)$$

Note that the signal to noise ratio (SNR) given here is a power ratio, yet we are given the SNR in decibels. We therefore need to convert back to a power ratio:

$$\begin{aligned}\text{SNR in dB} &= 10\log_{10}\left(\frac{S}{N}\right) \\ \text{therefore}\quad \frac{S}{N} &= 10^{(20/10)} \\ &= 100\end{aligned}$$

Therefore, the maximum data rate according to Shannon's theorem is:

$$\begin{aligned}\text{Max Data Rate} &= W\log_2\left(1+\frac{S}{N}\right) \\ &= 3000\times\log_2(1+100) \\ &\approx 20\text{kbps}\end{aligned}$$

The Nyquist limit for binary signalling over a 3kHz channel is

$$\begin{aligned}\text{Max Data Rate} &= 2W\log_2 M \\ &= 2\times 3000\times\log_2 2 \\ &= 6\text{kbps}\end{aligned}$$

Therefore, the maximum achievable data rate is 6kbps. (To achieve higher rates than this (up to the Shannon limit), one would have to use a different signalling method.)

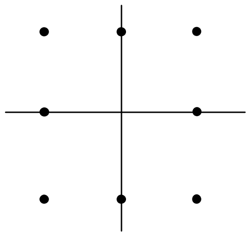
Question 3

Explain how analog signals are converted to digital signals?

The analog signal is first sampled (using PAM - Pulse Amplitude Modulation). Then the PCM (Pulse Code Modulation) technique is used. It assigns a value to each amplitude sample (sample is quantised) and encodes this value with n bit number.

Question 4

Two computers communicate via an analog channel using a signal modulated according to the following phase-amplitude diagram:



Assume the channel is divided into regular time slots each 20 microseconds wide.
(a) What kind of modulation is used?
(b) What is the baud-rate and the bit-rate?

(a) There are both phase shifts and amplitude changes, so it is phase-amplitude modulation.

(b) One signal lasts 20 microseconds, thefore:

$$\text{Baud rate} = \frac{1}{20 \times 10^{-6}} = 50000 \text{ baud}$$

$$\begin{aligned}\text{Bit rate} &= \text{baud rate} \times \text{bits per symbol}\\ &= 50000 \times \log_2 8\\ &= 150 \text{ kbps}\end{aligned}$$

Question 5

Ten signals, each requiring 4000 Hz, are multiplexed on to a single channel using FDM. How much minimum band-width is required for the multiplexed channel? Assume that the guard bands are 400 Hz wide.

There are ten 4000 Hz signals. We need nine guard bands to avoid any interference. The minimum bandwidth required is 4000*10+400*9=43,600 Hz.

Question 6

A cable company decides to provide Internet access over cable in a neighbourhood consisting of 5000 houses. The company uses a coaxial cable and spectrum allocation allowing 100 Mbps downstream bandwidth per cable. To attract customers, the company decides to guarantee at least 2 Mbps downstream bandwidth to each house at any time. Describe what the cable company needs to do to provide this guarantee.

A 2 Mbps downstream bandwidth guarantee to each house implies at most 50 houses per coaxial cable. Thus, the cable company will need to split up the existing cable into 100 coaxial cables and connect each of them directly to a fiber node.

Question 7

Assume the following parameters for a switching network:
N = number of hops between two given stations

L = message length, in bits
B = data rate, in bps, on all links
P = packet-size, in bits
H = overhead (header) bits per packet
S = call setup time (circuit switching or virtual circuit) in seconds
D = propagation delay per hop in seconds

(a) Derive general expressions for end-to-end delay for the four techniques: circuit switching, message switching, virtual circuit packet switching and datagram packet switching. Assume that there are no acknowledgements. (Note that in the lecture we did not introduce virtual circuit packet switching. This is illustrated on page 26 of part II of the lecture notes. "Datagram packet switching" is what we called packet switching in the lecture.)

(b) Using your results in (a), show under what conditions the delays are equal for
(i) circuit switching vs message switching, and
(ii) circuit switching vs (datagram) packet switching.

(a) (Note that we assume there is no queuing time.)

Circuit Switching

$$\begin{aligned} T &= \text{Call setup time + Message delivery time} \\ &= \text{Call setup time + (Propagation delay + Transmission time)} \\ &= S + N \times D + \frac{L}{B} \end{aligned}$$

Message switching

$$\begin{aligned} T &= N \times \text{Message delivery time for 1 hop} \\ &= N \times \left(D + \frac{L}{B}\right) \end{aligned}$$

Datagram packet switching

Datagram packet switching breaks the message into packets of size P (of which H bits are header information). The end-to-end delay is the time it takes to deliver the complete message through the first hop plus the time it takes to deliver the last packet through hop 2, hop 3, ... to the end.

Consider

$$D_1 = \text{Time to transmit and deliver all packets through first hop}$$
$$D_2 = \text{Time to transmit and deliver the last packet across one hop}$$
$$m = \text{Number of packets required to send message}$$

$$m = \frac{L}{(P-H)}$$
$$D_1 = m \times \frac{P}{B} + D$$
$$= \frac{L}{(P-H)} \times \frac{P}{B} + D$$
$$D_2 = \frac{P}{B} + D$$

$$T = D_1 + (N-1) \times D_2$$
$$= \frac{L}{(P-H)} \times \frac{P}{B} + D + (N-1)\left(\frac{P}{B} + D\right)$$

Virtual Circuit Packet Switching

$$T = \text{Call setup time + Datagram packet switching time}$$
$$= S + \frac{L}{(P-H)} \times \frac{P}{B} + D + (N-1)\left(\frac{P}{B} + D\right)$$

(b) Circuit switching vs Message switching

The condition can be derived by equating the delay expressions determined above:

$$\text{Circuit switching delay} = \text{Message switching delay}$$
$$S + N \times D + \frac{L}{B} = N \times \left(D + \frac{L}{B}\right)$$
$$S \times B = (N-1) \times L$$

Circuit switching vs datagram packet switching

$$\text{Circuit switching delay} = \text{Datagram packet switching delay}$$
$$S + N \times D + \frac{L}{B} = \frac{L}{(P-H)} \times \frac{P}{B} + D + (N-1)\left(\frac{P}{B} + D\right)$$
$$S + N \times D + \frac{L}{B} = \frac{L}{(P-H)} \cdot \frac{P}{B} + D + (N-1) \cdot \frac{P}{B} + (N-1)D$$
$$S + ND + \frac{L}{B} = \left(\frac{L}{P-H} + N - 1\right)\frac{P}{B} + ND$$
$$S + \frac{L}{B} = \left(\frac{L}{P-H} + N - 1\right)\frac{P}{B}$$

Question 8

Compare the delay in sending an x-bit message over a k-hop path in a circuit-switched network and in a (lightly loaded) packet switch network. The circuit set-up time is s seconds, the propagation delay is d seconds per hop, the packet size is p-bits, and the data rate is b bps. Under what conditions does the packet network have a lower delay?

We assume that the header is negligible compared with the rest of the packet.

Delay for circuit switching: $T_c = s + \frac{x}{b} + k \times d$

Delay for packet switching: $T_p = \frac{x}{b} + (k-1)\frac{p}{b} + k \times d$

Packet switching is faster (i.e. lower delay) if $T_p < T_C$, that is: $s > (k-1)\frac{p}{b}$.